PERGAMON

International Journal of Solids and Structures 36 (1999) 3427–3441

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

Analysis of compressive failure of layered materials by kink band broadening

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Received 13 August 1997; in revised form 27 April 1998

Abstract

Failure by steady state kink band broadening in uni-directional fibre composites or layered materials is analysed. An incremental scheme for calculation of kink band broadening stresses and lock-up conditions in the band for arbitrary material behaviour is presented. The method is illustrated by material data which are representative for polymer matrix composites for which experimental work exists. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

Fail safe design of materials and structures often involves design against propagating instabilities. Such instabilities propagate under steady state conditions at characteristic stresses in the material, or characteristic loads on the structure, and may thus lead to general failure if they cannot be arrested. Examples of propagating instabilities in structures involve bulging of internally pressurized rubber cylinders and propagating buckles in undersea pipelines (e.g. Palmer and Martin, 1975; Chater and Hutchinson, 1984; Jensen, 1988). Examples of propagating instabilities on the material scale involve neck propagation in drawing of polymers (e.g. Hutchinson and Neale, 1983), channelling and tunnelling cracks in layered materials (e.g. Hutchinson and Suo, 1992; Thouless et al., 1992; Jensen, 1994), propagating zones of phase transformations, and crushing of foams. The review article of Kyriakides (1994) describes analyses of some of the above mentioned propagating instabilities.

Recently, a failure mechanism of the type as above namely kink band broadening was described by Moran et al. (1995) for a fibre composite under compression parallel to the fibres. Kink band formation in fibre composites with a polymer matrix has for a long time been recognised as an

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PII: S0020–7683(98)00158–9

important failure mechanism under compressive stresses (Budiansky, 1983). Common to the other propagating instabilities mentioned is that the stress required to initiate the kink band is significantly higher than the steady state kink band broadening stress at which the instability propagates through the material. Furthermore, the initiation stress is sensitive to imperfections, notches, etc., while the steady state kink band broadening stress is a constant. Thus, the full load bearing capacity of the material may not be exploited fully when designing for safety against band broadening. The resulting material or structure, however, should be quite tolerant to damage and imperfections.

Most of the theoretical work on failure of fibre composites in compression due to fibre kinking has been concerned with aspects of the initiation of kink bands. Kink band propagation transverse to the fibres was studied in Fleck and Budiansky (1991), while kink band broadening was studied theoretically in Moran et al. (1995), Liu et al. (1996), Fleck (1997) and Budiansky et al. (1997) and experimentally in Moran et al. (1995), Liu et al. (1996), Poulsen et al. (1997) and Vogler and Kyriakides (1997).

In the previous theoretical studies of kink band broadening, a simplified kinematical description of the deformation in the composite is employed by assuming infinitely rigid fibres. Furthermore, in the previous studies a priori assumptions based on experimental observations are made concerning the state of deformation inside and outside the kink band.

In the present analysis of quasi-static, steady state kink band broadening, arbitrary time-independent behaviour of all composite constituents is allowed for, and the states of deformation inside as well as outside the kink band are determined solely by equilibrium, continuity, and balance of work.

2. Constitutive equations

The geometry of the composite considered in this work is shown in Fig. 1. The model is two dimensional and plane strain conditions are assumed. Thus, the model applies to layered materials and, in an approximate manner, also to uni-directional fibre composites. This is reflected in the following where the constituents are referred to both as layers or fibres and matrix. The use of a plane description of a fibre composite is in line with most theoretical work on fibre kinking. Attempts have been made, though, to incorporate three-dimensional effects in the models, see for instance Gradidier et al. (1992). The present model is applied to a composite with only two

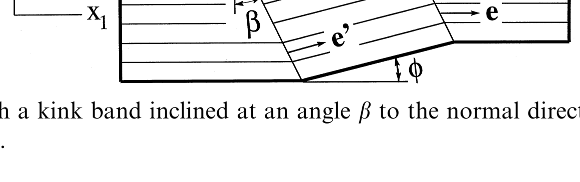

Fig. 1. Periodic, composite with a kink band inclined at an angle β to the normal direction of the fibres. The fibres in the band are rotated at angle ϕ.

constituents which alternate periodically. The formulation, however, is general and applies to both non-periodic and multi-phase composites.

2.1. Constituent behaviour

Three constitutive models for the composite constituents have been compared; the Hutchinson and Neale (1978) version of J_2-deformation theory, the Stören and Rice (1975) version of J_2-deformation theory, and the J_2-flow theory (McMeeking and Rice, 1975).

All constituents are allowed to behave non-linearly as long as they are described by time-independent elastic–plastic constitutive relations. It was found experimentally in Kyriakides et al. (1995) for a carbon fibre reinforced polymer matrix composite that the fibres responded non-linearly upon compression to strain levels relevant for kink band development. This non-linearity was taken to be entirely due to the material behaviour as opposed to geometrical effects.

The tensor of elastic–plastic tangent moduli, L_{ijkl}, relating Jaumann rates of Kirchhoff stresses, $\hat{\tau}_{ij}$, to strain rates, ε_{ij}, on the form

$$\hat{\tau}_{ij} = L_{ijkl}\varepsilon_{lk} \tag{1}$$

will, in the following, be presented in two of the three cases considered. In terms of velocity gradients, $v_{i,j}$, the strain rates are given by

$$\varepsilon_{ij} = \tfrac{1}{2}(v_{i,j}+v_{j,i}) \tag{2}$$

The J_2-deformation theory of Hutchinson and Neale (1978) has been applied in Jensen and Christoffersen (1997) and the theory is outlined there. For this reason, the expressions for the moduli will not be repeated here. For completeness, however, the expressions for the moduli are listed below for the two other theories.

The components of the tensor of instantaneous moduli in the case of the J_2-deformation theory of Stören and Rice (1975) are given by

$$L_{ijkl} = G_s(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})+(K-\tfrac{2}{3}G_s)\delta_{ij}\delta_{kl}-\tfrac{4}{3}(G_s-G_t)m_{ij}m_{kl} \tag{3}$$

where δ_{ij} denotes the Kroecker delta, and m_{ij} is given in terms of Cauchy stresses, σ_{ij}, by

$$m_{ij} = \frac{1}{2\sigma_{eq}}\left(\sigma_{ij}-\frac{1}{3}\delta_{ij}\sigma_{kk}\right) \tag{4}$$

Here, σ_{eq} is the equivalent von Mises stress

$$\sigma_{eq} = \sqrt{\tfrac{3}{2}\sigma_{ij}\sigma_{ij}-\tfrac{1}{2}\sigma_{ii}\sigma_{jj}} \tag{5}$$

Furthermore, K is the bulk modulus, and G_s and G_t are, respectively, the secant and the tangent shear moduli. The relationship between these moduli and the secant and tangent moduli, E_s and E_t, with Poisson's ratio, v, and Young's modulus, E, included, is

$$\frac{1}{G_s} = \frac{3}{E_s}-\frac{1-2v}{E} \qquad \frac{1}{G_t} = \frac{3}{E_t}-\frac{1-2v}{E} \qquad \frac{1}{K} = \frac{3(1-2v)}{E} \tag{6}$$

A Ramberg–Osgood relation for the natural (logarithmic) strain, e, vs uniaxial stress, σ, curve is assumed

$$e = \frac{\sigma}{E} + \frac{3\sigma^y}{7E}\left(\frac{\sigma}{\sigma^y}\right)^n \tag{7}$$

Here, σ_y is a reference stress, and n is the hardening index.

In J_2-flow theory, the components of the tensor of instantaneous moduli are given by (McMeeking and Rice, 1975)

$$L_{ijkl} = G(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + (K - \tfrac{2}{3}G)\delta_{ij}\delta_{kl} - \tfrac{4}{3}(G - G_t)m_{ij}m_{kl} \tag{8}$$

where G is the elastic shear modulus and the parameters G_t and K are chosen according to (6). Isotropic hardening is assumed.

The case of unloading is, in the J_2-flow theory, represented by the condition

$$G_t = G \quad \text{for } m_{ij}\varepsilon_{ij} \leqslant 0 \tag{9}$$

2.2. Composite behaviour

The homogenization procedure of Christoffersen and Jensen (1996) and Jensen and Christoffersen (1997) for non-linear, layered composites is employed for extracting the overall composite moduli. The constitutive eqns (1) are written in the form

$$\dot{s}_{ij} = C_{ijkl}v_{l,k} \tag{10}$$

where $\dot{s}_{ij}$ are the nominal stress rates. The relationship between tensors L_{ijkl} and C_{ijkl} is given by

$$C_{ijkl} = L_{ijkl} - \tfrac{1}{2}\delta_{il}\sigma_{kj} - \tfrac{1}{2}\delta_{ik}\sigma_{lj} - \tfrac{1}{2}\sigma_{il}\delta_{kj} + \tfrac{1}{2}\sigma_{ik}\delta_{lj} \tag{11}$$

Two Cartesian coordinate systems are introduced (Fig. 1). The x_1-axes are aligned with the fibres, and the strain rates are taken to be piecewise homogeneous in each constituent. By equilibrium and compatibility, it was shown in Christoffersen and Jensen (1996) that the composite moduli, C_{ijkl}, could be expressed in terms of the fibre and matrix moduli, C^{f}_{ijkl} and C^{m}_{ijkl}, and volume fractions, c^{f} and c^{m}, as

$$\mathbf{C}_{\alpha\beta} = c^{\mathrm{f}}\mathbf{C}^{\mathrm{f}}_{\alpha\beta} + c^{\mathrm{m}}\mathbf{C}^{\mathrm{m}}_{\alpha\beta} - c^{\mathrm{f}}c^{\mathrm{m}}(\mathbf{C}^{\mathrm{f}}_{\alpha 2} - \mathbf{C}^{\mathrm{m}}_{\alpha 2})\mathbf{C}^{*-1}_{22}(\mathbf{C}^{\mathrm{f}}_{2\beta} - \mathbf{C}^{\mathrm{m}}_{2\beta}) \tag{12}$$

where $\mathbf{C}^*_{22}$ denotes the matrix

$$\mathbf{C}^*_{22} = c^{\mathrm{m}}\mathbf{C}^{\mathrm{f}}_{22} + c^{\mathrm{f}}\mathbf{C}^{\mathrm{m}}_{22} \tag{13}$$

and $\mathbf{C}_{\alpha\beta}$ represents the matrices

$$\mathbf{C}_{11} = \begin{pmatrix} C_{1111} & C_{1112} \\ C_{1211} & C_{1212} \end{pmatrix} \quad \mathbf{C}_{12} = \begin{pmatrix} C_{1121} & C_{1122} \\ C_{1221} & C_{1222} \end{pmatrix}$$

$$\mathbf{C}_{21} = \begin{pmatrix} C_{2111} & C_{2112} \\ C_{2211} & C_{2212} \end{pmatrix} \quad \mathbf{C}_{22} = \begin{pmatrix} C_{2121} & C_{2122} \\ C_{2221} & C_{2222} \end{pmatrix} \tag{14}$$

i.e. matrix $\mathbf{C}_{\alpha\beta}$ has $C_{\alpha i\beta j}$ as element (i,j). The representation of the composite moduli in a matrix notation rather than in tensor notation was found to be convenient in Christoffersen and Jensen (1996) and Jensen and Christoffersen (1997).

3. Kink band development

In this section, the equilibrium and continuity conditions across a straight kink band boundary (see Fig. 1) are reviewed. A numerical procedure is formulated in which an imperfection in the composite is assumed to exist in the form of a kink band. In this band, the fibres are initially rotated an angle, ϕ_0. Increments of fibre rotation in the kink band are specified in a numerical procedure where local stresses and velocities in the fibres and the matrix inside and outside the kink band are related through the equations listed below. The overall stress state in the composite is homogeneous inside and outside the band. In the base material, outside the straight kink band,

$$C_{ijkl}v_{l,k}-\sigma_{ij}v_{k,k}+v_{i,k}\sigma_{kj}=\dot{\sigma}_{ij} \tag{15}$$

where $\dot{\sigma}_{ij}$ or $v_{i,j}$ or combinations of these are given by the external loading and the boundary conditions. Results in the following are presented for $\sigma_{12}=0$ and $\sigma_{22}=0$. A quantity $\dot{(\,)}$ denotes an increment.

State variables inside the kink band are extracted by considering equilibrium and continuity across the band boundary. As state variables are referred to Cartesian coordinate systems with their x_1-axes aligned with the fibres (Fig. 1),

$$v_{2,1}=0 \quad v'_{2,1}=0 \tag{16}$$

where primed symbols pertain to the base material, and non-primed symbols to the kink band. The first condition ties down a rigid body rotation, the other condition represents a rotation of the band coordinate system relative to the base material system.

Continuity of velocities across the boundary between base material and kink band requires that

$$w'_{i,j}t'_jt'_i=v_{i,j}t_jt_i \quad w'_{i,j}t'_jn'_i=v_{i,j}t_jn_i \tag{17}$$

where

$$w'_{i,j}=v'_{i,j}+\Omega_{ij} \quad \Omega_{21}=-\Omega_{12}=\dot{\phi} \quad \Omega_{11}=\Omega_{22}=0 \tag{18}$$

In (17), n_i and n'_i, t_i and t'_i are, respectively, the unit normal and the unit tangent of the boundary expressed, as appropriate, in the base material coordinates or the kink band coordinates. In (18) the quantity $\dot{\phi}$ is the relative, incremental rotation of the coordinate systems cf Fig. 1.

Continuity of traction rates across the boundary between the two regions requires that

$$n'_iC'_{ijkl}w'_{l,k}n'_j=n_iC_{ijkl}v_{l,k}n_j \quad n'_iC'_{ijkl}w'_{l,k}t'_j=n_iC_{ijkl}v_{l,k}t_j \tag{19}$$

given the tractions are themselves continuous.

The kink band inclination, β, and fibre and matrix densities change with deformation according to

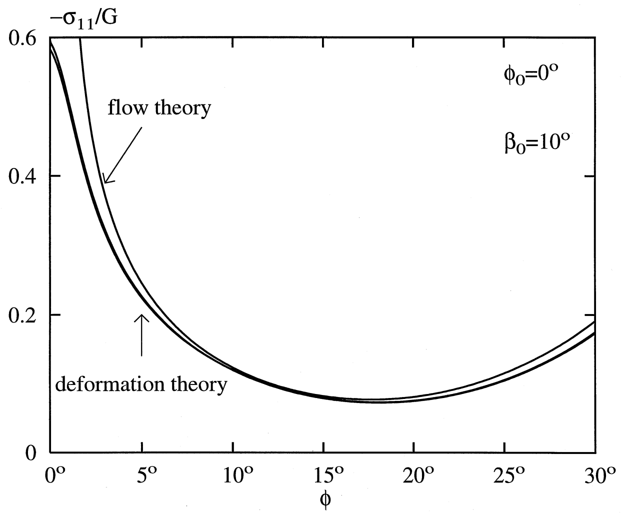

Fig. 2. Kink stress, $-\sigma_{11}$, normalised by the elastic shear modulus of the composite, G, as a function of fibre rotation, ϕ, for three constitutive laws, small initial imperfection.

$$\dot{\beta} = -v_{i,j}t_jn_i \quad n_i'e_i'\frac{c'^{\mathrm{f}}}{A'^{\mathrm{f}}} = n_ie_i\frac{c^{\mathrm{f}}}{A^{\mathrm{f}}} \tag{20}$$

where e_i is a unit vector parallel with the fibres. Initial values for the kink band inclination, β_0, and volume fractions are assumed and updated in the incremental numerical formulation. Note, that volume fractions and areas are different inside and outside the kink band as the stress states differ. With $A^{\mathrm{f}}/c^{\mathrm{f}} = A^{\mathrm{m}}/c^{\mathrm{m}}$, $\dot{A}^{\mathrm{f}}/A^{\mathrm{f}} = v^{\mathrm{f}}_{2,2}$ and $\dot{A}^{\mathrm{m}}/A^{\mathrm{m}} = v^{\mathrm{m}}_{2,2}$

$$\dot{c}^{\mathrm{f}} = c^{\mathrm{f}}c^{\mathrm{m}}(v^{\mathrm{f}}_{2,2} - v^{\mathrm{m}}_{2,2}) \tag{21}$$

Volume fractions outside the kink band are updated according to (21), while volume fractions and areas inside the kink band are given by (21) and (20) utilizing the conditions for overall equilibrium.

In each step of the incremental procedure, stresses in the fibres and the matrix inside and outside the kink band are updated, and the corresponding moduli are given by the relations in Section 2. As the magnitude of the initial imperfection is reduced towards zero, the response of the imperfect composite approaches that of a perfect composite where the kink band forms at a bifurcation stress (Jensen and Christoffersen, 1997).

In Figs 2 and 3, results based on the numerical procedure are presented. Comparisons of the responses are given for a small initial imperfection, $\phi_0 \approx 0^\circ$ (Fig. 2) and a realistic imperfection, $\phi_0 = 3^\circ$ (Fig. 3). The constitutive relations which are compared are the Stören and Rice deformation theory, the J_2-flow theory, and the Hutchinson and Neale deformation theory. The initial kink band inclination is taken to be $\beta_0 = 10^\circ$. The volume fraction of fibres is taken to be

$$c^{\mathrm{f}} = 0.6 \tag{22}$$

The elastic parameters for the fibres and the matrix are

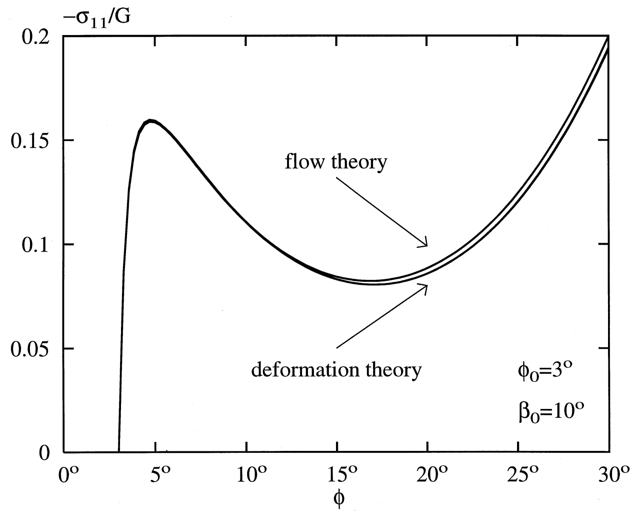

Fig. 3. Kink stress, $-\sigma_{11}$, normalised by the elastic shear modulus of the composite, G, as a function of fibre rotation, ϕ, for three constitutive laws, large initial imperfection.

$$E^{\mathrm{f}}/E^{\mathrm{m}} = 35 \quad v^{\mathrm{f}} = 0.263 \quad v^{\mathrm{m}} = 0.356 \tag{23}$$

The fibres are assumed to be linear elastic, while the parameters in the Ramberg–Osgood relation (7) for the matrix are taken to be

$$\sigma^{y,\mathrm{m}}/E^{\mathrm{m}} = 0.013 \quad n^{\mathrm{m}} = 4 \tag{24}$$

These parameters model the APC-2/AS4 composite investigated experimentally in Kyriakides et al. (1995) and Vogler and Kyriakides (1997).

By Figs 2 and 3, the predictions based on the two deformation theories are nearly identical for small and large imperfections and for large fibre rotations. The Hutchinson and Neale version of the large strain J_2-deformation theory is a true path independent theory (Hutchinson and Neale, 1981), while the Stören and Rice version is computationally more efficient.

It is also seen that the differences between the deformation theory predictions and the flow theory predictions are significant only for small imperfections and small fibre rotations. This is due to the different bifurcation predictions for the J_2-flow theory and the J_2-deformation theory which are well known in numerous other plastic bifurcation problems (Hutchinson, 1974). For realistic imperfections and large fibre rotations and deformation and flow theory predictions are nearly identical. This indicates that path dependent effects play a minor role for the present problem.

For the reasons mentioned above, all predictions in the following are based on the Stören and Rice deformation theory outlined in Section 2.

4. Steady-state kink band broadening

A scenario of steady state kink band broadening in a layered material under uniaxial compression is sketched in Fig. 4. If a composite is loaded by a compressive stress, the overall stresses will

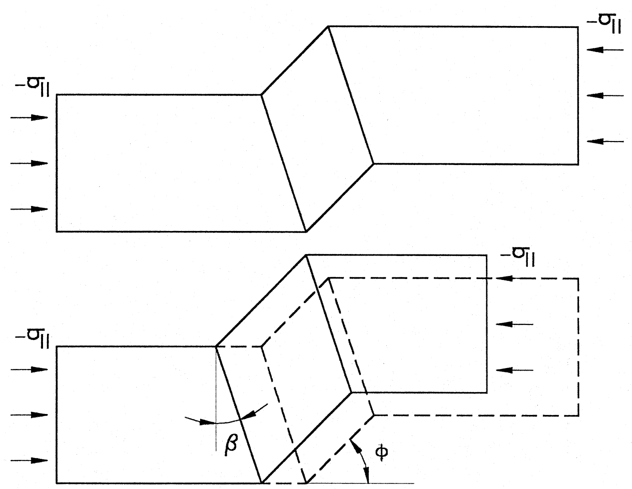

Fig. 4. Sketch of steady-state kink band broadening at constant external stress and fixed strains well outside and inside the band.

remain homogeneous until the applied stress reaches a critical value where the strains localise and a kink band forms with the geometric definitions given in Fig. 1. After the initiation, the kink band broadens as shown in Fig. 4 under a constant applied stress so that the boundary between kinked and unkinked material propagates at steady state in a direction parallel to the fibres. The strains in the material far ahead of, and far behind, the propagating front do not change. In the kinked material, the deformations are determined by the so called lock-up conditions where the fibres stop rotating.

The previous models on kink band broadening (Moran et al., 1995; Fleck, 1997; Budiansky et al., 1997) have been based on the assumption that the fibres are infinitely rigid which simplifies the kinematical description of the composite deformations considerably. More important, however, is the assumptions of the lock-up conditions applied in the previous works. Motivated by experimental studies (e.g. Evans and Adler, 1978) the condition for lock-up applied in the previous works is

$$\phi = 2\beta \tag{25}$$

corresponding to zero volumetric straining of the matrix material between the rigid fibres. The condition for fibre lock-up was applied also in Fleck and Budiansky (1991) based on the idea that the composite resists compressive transverse straining in a highly stiff manner. Experimental observations also exist, however, indicating that (25) is not satisfied very precisely (Poulsen et al., 1997; Vogler and Kyriakides, 1997).

The present model differs from the previous models in several aspects. The kinematics is not based on assuming inextensional fibres, and, more crucially no a priori assumptions are made concerning the conditions for fibre lock-up in the kink band or initial conditions outside the kink band. In the present formulation, the lock-up condition follows from energy balance for the work done by external loads as the kink band broadens and the work done by local stresses in the band as the fibres rotate from initial to final state.

The work done per unit volume by the stresses in the kink band is

$$W^{\mathrm{I}} = \int_{\underline{\varepsilon}}^{\underline{\varepsilon}'} \tau'_{ij}\, \mathrm{d}\varepsilon'_{ij} \tag{26}$$

where $\underline{\varepsilon}$ and $\underline{\varepsilon}'$ are, respectively, the strain states outside and inside the kink band, both treated as unknowns.

The work done per unit volume by the external loads as the kink band broadens is

$$W^{\mathrm{E}} = t_{ij}\Delta\varepsilon^{*}_{ij} \tag{27}$$

where $\Delta\varepsilon^{*}_{ij}$ denotes the difference between the Lagrangian strain state inside and outside the kink band expressed in the base material coordinate system, and t_{ij} denotes the work conjugate, external stresses (i.e. the second Piola–Kirchhoff stresses) obtained in the updating procedure through relations that can be found in McMeeking and Rice (1975).

It has been observed (Moran et al., 1995) that once a kink band has been formed and broadens in a steady state manner, fibres outside the kink band far ahead of the advancing kink band front and fibres in the kink band far behind the advancing front are straight. Provided the constitutive behaviour of the constituents is described by a deformation theory, the work balance

$$W^{\mathrm{I}} = W^{\mathrm{E}} \tag{28}$$

can be evaluated by a deformation history where no fibre bending occurs, even-though the actual behaviour observed involves bending of the fibres in the vicinity of the advancing front due to the load-path independence. If the constituents are described by constitutive relations where the work is dependent on the deformation path such as the J_2-flow theory, the actual path should be traced when evaluating (28).

In the incremental numerical procedure presented in Section 3 where the development of a kink band is traced by prescribing the fibre rotation in the band and updating the geometry, the stresses, and the moduli, the energy balance (28) is checked at each increment. Lock-up by definition occurs when (28) is satisfied.

The kinematics simplifies considerably if the fibres are assumed to be rigid as in the previous studies. In this limit, Moran et al. (1995), Fleck (1997) and Budiansky et al. (1997) replaces (27) by

$$W^{\mathrm{E}} = -\sigma_{11}(1-\cos\phi) \tag{29}$$

for uniaxial loading. In the present study (27) rather than (29) is used for W^{E}. In Fig. 5, results for W^{I} based on (26) and W^{E} based on (27) and (29) are shown as a function of fibre rotation, ϕ. The material parameters are given by (22), (23) and (24), and the initial kink band inclination is set to $\beta_0 = 10^{\circ}$. The lock-up condition is marked as the intersection of the W^{I} and W^{E} curves. It should be noted that for these material parameters, (29) is an accurate approximation to (27), but the lock-up condition (25) still deviates considerably from the fibre rotation at lock-up in Fig. 5. In Fig. 6, a calculation is shown with similar material parameters except for (23) where $E^{\mathrm{f}}/E^{\mathrm{m}} = 35$

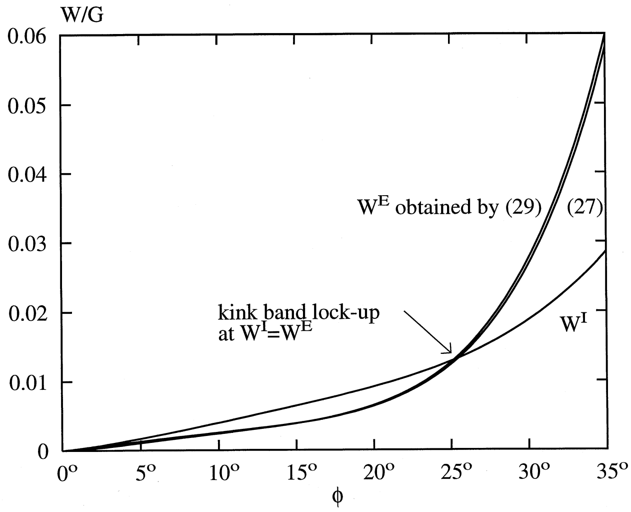

Fig. 5. Internal and external work, W^{I} and W^{E}, normalised by the elastic shear modulus of the composite, G, as a function of fibre rotation, ϕ. Material data given by (22)–(24), $\beta_0 = 10^\circ$. Internal work is calculated by (26) while external work is calculated by (27) with (29) included for comparison. Kink band lock-up is determined by the work balance condition $W^{\mathrm{I}} = W^{\mathrm{E}}$.

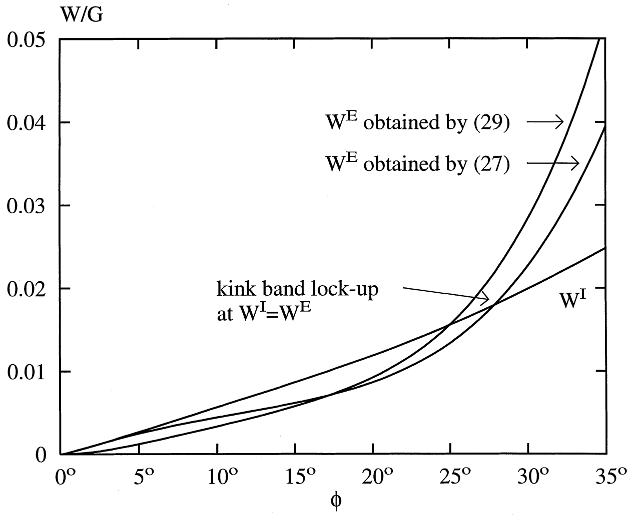

Fig. 6. Internal and external work, W^{I} and W^{E}, normalised by the elastic shear modulus of the composite, G, as a function of fibre rotation, ϕ. Material data given by (22)–(24), except for $E^{\mathrm{f}}/E^{\mathrm{m}} = 3.5$, $\beta_0 = 10^\circ$. Internal work is calculated by (26) while external work is calculated by (27) with (29) included for comparison. Kink band lock-up is determined by the work balance condition $W^{\mathrm{I}} = W^{\mathrm{E}}$.

is replaced by $E^f/E^m = 3.5$ in order to illustrate the influence of finite fibre stiffness on the accuracy of (29).

As can be seen by Fig. 5, (29) can be expected to be a reasonable approximation to (27) for typical polymer matrix composites. So the difference between the present and the previous analyses is introduced in the expression for W^I (26). Since the condition (25), in which the matrix is in a state of pure shear, is enforced as the lock-up condition in the previous studies, and since the strains outside the kink band are zero for a uniaxial loading parallel to the rigid fibres, the work (26) can be evaluated through a strain history that only involves shear. In the present interpretation, this approach ignores the contribution to (26) due to the work done by transverse stresses on the transverse strains in the kink band as (25) is not enforced here.

In Fig. 7, the development with fibre rotation of shear stresses in the matrix, σ^m_{12}, inside the kink band is shown for three values of initial kink band inclination, β_0. The material parameters are given by (22)–(24). The relationship between matrix shear stress and fibre rotations in Fig. 7 resembles the tri-linear curves assumed in Moran et al. (1995) for the pure shear stress vs shear strain response. For the present analysis it is emphasised that the behaviour in Fig. 7 is a result of multiaxial effects; the pure shear stress vs shear strain relation would resemble the Ramberg–Osgood relation (7) for uniaxial loading. In Moran et al. (1995) closed form expressions for band broadening stresses etc. were obtained as functions of parameters in the tri-linear shear response that they assumed and by comparison with Fig. 7 it would be possible to relate these parameters to the uniaxial behaviour, the initial kink band inclination, etc., however, this issue is not pursued further here.

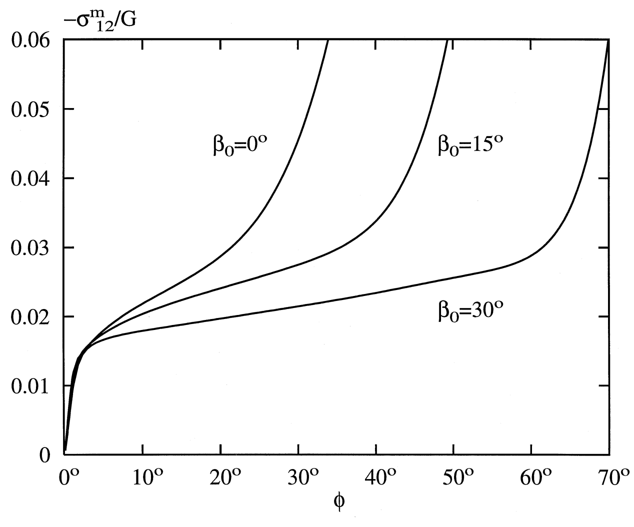

Fig. 7. Shear stresses in the matrix inside the kink band, σ^m_{12}, normalised by the elastic shear modulus for the composite, G, as a function of fibre rotation, ϕ. Results are shown for three values of initial kink band inclination; $\beta_0 = 0^\circ$, $\beta_0 = 15^\circ$ and $\beta_0 = 30^\circ$.

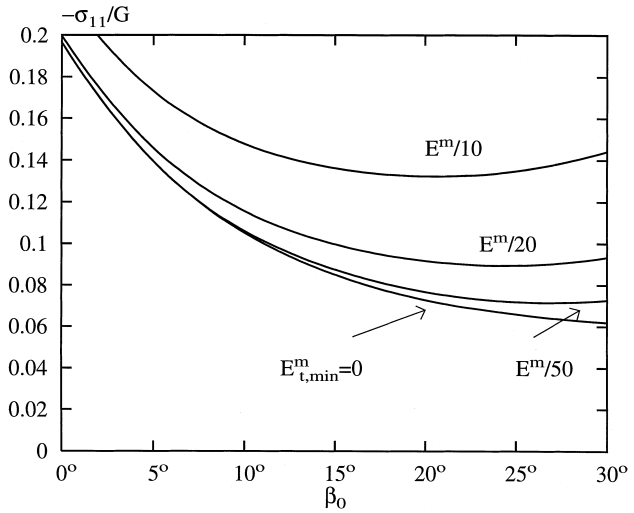

Fig. 8. Kink band broadening stress, σ_{11}, normalised by the elastic shear modulus for the composite, G, for different initial kink band inclinations, β_0. Material data given by (22)–(24), with tangent modulus for the matrix reduced at most to $E^{m}_{t,min}$.

5. Results and discussion

The kink band broadening stress and the fibre rotation at lock-up obtained by the numerical procedure described in the previous section is now found for different values of initial kink band inclination, β_0. The material parameters are given by (22), (23) and (24), and the results for the stress are shown as the lowest curve in Fig. 8. It turns out that the kink band broadening stress is quite sensitive to the uniaxial stress strain curve assumed for the matrix. Especially the uniaxial behaviour at large strains seems to be critical. The Ramberg–Osgood relation for the matrix (7) with the parameters (24) is a good approximation to the measured curves in Kyriakides et al. (1995) for strains up to approximately 5%. Unfortunately, only little information is available for larger strains (see e.g. G'Sell et al., 1990), however, it seems that a Ramberg–Osgood relation fitted to the small strain regime does not extrapolate well into the large strain regime. In order to illustrate that the uniaxial response of the matrix at large strains is important, three calculations are included in Fig. 8. These calculations are, again, based on the material parameters (22)–(24), but now with the exception that the Ramberg–Osgood relation (7) is used for the matrix until the tangent modulus, E^{m}_{t}, is reduced to some specified fraction, $E^{m}_{t,min}$ of the linear elastic modulus, E^{m}. From that point, the uniaxial curve is approximated by a straight line with $E^{m}_{t} = E^{m}_{t,min}$. The three curves included in Fig. 8 are based on $E^{m}_{t,min} = E^{m}/50$, $E^{m}/20$ and $E^{m}/10$ all of which agree with the measurements of Kyriakides et al. (1995) in the small strain regime.

The fibre rotations at kink band lock-up are shown in Fig. 9 as a function of initial kink band inclination, β_0, for the case $E^{m}_{t,min} = E^{m}/20$. The relation (25) with $\beta = \beta_0$ is included for comparison. This relation is applied in previous studies, and it is seen that (25) underestimates the present fibre rotations at lock-up. This is consistent with the observations of Vogler and Kyriakides (1997) and Poulsen et al. (1997).

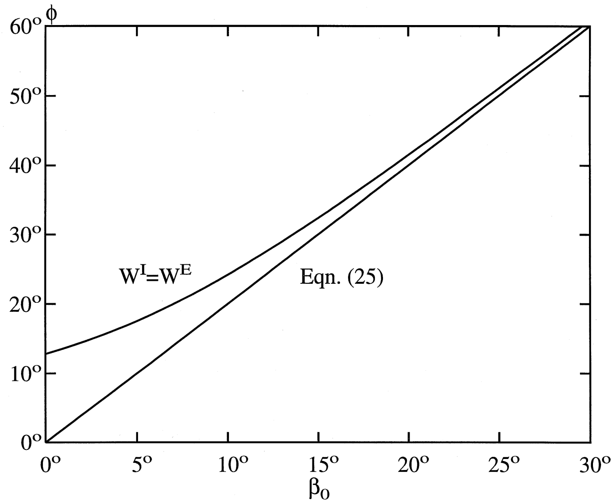

Fig. 9. Fibre rotation at kink band lock-up determined by the present work balance condition as a function of initial band inclination, β_0. The condition (25) used in previous works is included for comparison.

The steady-state kink band broadening stress could be thought of as the minimum value of $-\sigma_{11}$ as a function of β_0 (Moran et al., 1995; Liu et al., 1996), thus defining also the most critical band inclination. Alternatively, Fleck (1997) assumes that the kink band inclination is set by other details of the kink band propagation process so that the steady state kink band broadening stress is not necessarily the minimum value of $-\sigma_{11}$ as a function of β_0. The measured values in Vogler and Kyriakides (1997) was approximately $\beta_0 = 15^\circ$, and $-\sigma_{11}/G = 0.085$. By comparison of these measured values and the results in Figs 8 and 9, it is seen that reasonable agreement can be obtained. It is also clear that further measurements of the matrix behaviour at large strains are important for the modelling of kink band broadening. Values of β_0 around 22–23° were observed in Moran et al. (1995) for a carbon/PEEK composite and also in Poulsen et al. (1997) for wood.

Budiansky et al. (1997) have discussed the possibility suggested by Moran et al. (1995) of using the steady-state kink band broadening stress as a rational working stress for design purposes. This has the advantage that the kink band broadening stress is a constant determined by the properties of the constituents in the composite, while the kink band initiation stress is imperfection sensitive (cf Figs 2 and 3). This may be unduly conservative, however, as the band broadening stress can be as low as one third the kink band initiation stress for material parameters realistic for polymer matrix composites. This discussion resembles that for a structural instability mentioned in the introduction; propagating buckles in undersea pipelines driven by external pressure. There, also, the propagation pressure is significantly lower than the pressure required to initiate the pipeline buckling, and now pipeline designers seem to use the initiation pressure rather than the propagation pressure for design purposes. This also has to do with the fact that propagating buckles can be arrested by mounting devices on the pipeline, with the fact that the initiation pressure is much less sensitive to imperfections than the kink band initiation stress in fibre composites, and with the fact that the pipeline can be protected from damage in service. On the other hand it is known, that if a layered material is designed so that tunnel cracks (Hutchinson and Suo, 1992) cannot propagate, the resulting material is highly damage tolerant. To return to the present problem in the light of the above, whether or not the kink band broadening stress should be used for design purposes depends on how critical a propagating kink band is for the given structural component and how well controlled the imperfections due to fabrication and in-service loads are.

6. Conclusion

Failure by kink band broadening in layered materials has been analysed. A general framework for numerical calculation of kink band broadening stresses and fibre rotations in the kink band have been formulated. Results have been presented for uniaxial compression of a periodic bi-layer composite. It is shown that three large strain constitutive relations result in nearly identical predictions. Material parameters are chosen for illustration to model a polymer matrix composite with relatively stiff fibres compared to the matrix. It is shown that good agreement between calculations and experiments can be obtained, but also that the large strain response of the matrix is critical for kink band broadening in polymer matrix composites. Work is currently in progress to model effects of damage typically occurring in such composites at large strains.

The present analysis deviates from previous ones in several aspects. The formulation allows for arbitrary time independent plasticity of general, layered materials. Furthermore, in contrast to previous analyses, no assumptions based on experimental observations are made concerning the conditions for fibre lock-up in the kink band. The lock-up condition applied in the present work is balance of work done by stresses in the kink band as the fibres rotate and the work done by external stresses as the kink band broadens. As a consequence in the present interpretation, the contribution to the work done by transverse stresses on transverse strains in the kink band has been ignored in previous studies.

For the cases considered, the fibre rotations at lock-up exceeds the rotations assumed in previous studies which is consistent with experimental observations.

Acknowledgements

The work is supported by the MUP2 Center for Materials Processing, Properties, and Modeling, financed by The Danish Agency for Development of Trade and Industry, The Danish Natural Science Research Council, and The Danish Technical Research Council.

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